MODEL ANSWERS

AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level
MATHEMATICS
Paper 3

Friday 15 June 2018 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

Answer **all** questions in the spaces provided.

1 A circle has equation $(x-4)^2+(y+4)^2=9$

What is the area of the circle?

Radius $= \sqrt{9} = 3$
Area $= \pi r^2 = \pi(3)^2 = 9\pi$

Circle your answer.

[1 mark]

3π $\quad$ 9π $\quad$ 16π $\quad$ 81π

2 A curve has equation $y = x^5 + 4x^3 + 7x + q$ where q is a positive constant.

Find the gradient of the curve at the point where $x = 0$

Circle your answer. $\frac{dy}{dx} = 5x^4 + 12x^2 + 7$, at $x=0$: $\frac{dy}{dx} = 7$

[1 mark]

0 $\quad$ 4 $\quad$ 7 $\quad$ q

3 The line L has equation $2x + 3y = 7$

Which one of the following is perpendicular to L?

Tick **one** box.

[1 mark]

$2x - 3y = 7$ ☐
$y = \frac{2}{3}x - \frac{7}{3}$

$3x + 2y = -7$ ☐
$y = -\frac{3}{2}x - \frac{7}{2}$

$2x + 3y = -\frac{1}{7}$ ☐
$y = -\frac{2}{3}x - \frac{1}{21}$

$3x - 2y = 7$ ☑
$y = \frac{3}{2}x - \frac{7}{2}$

$2x + 3y = 7$
$y = -\frac{2}{3}x + \frac{7}{3}$
Gradient $= -\frac{2}{3}$

So we are looking for a line with gradient $\frac{-1}{(-2/3)} = \frac{3}{2}$.

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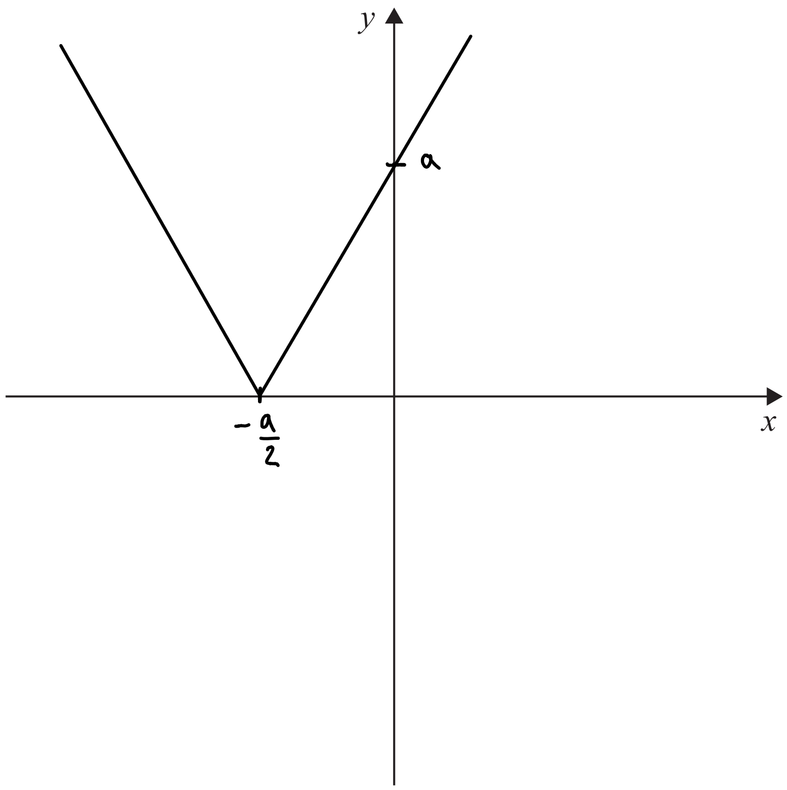
4 Sketch the graph of $y = |2x + a|$, where a is a positive constant.

Show clearly where the graph intersects the axes.

[3 marks]



5 Show that, for small values of x, the graph of $y = 5 + 4\sin\frac{x}{2} + 12\tan\frac{x}{3}$ can be approximated by a straight line.

[3 marks]

$y = 5 + 4\sin\left(\frac{x}{2}\right) + 12\tan\left(\frac{x}{3}\right)$

For small x: $\sin x \approx x$

$\tan x \approx x$

So, for small x: $y \approx 5 + 4\left(\frac{x}{2}\right) + 12\left(\frac{x}{3}\right)$

$y \approx 5 + 2x + 4x$

$y \approx 6x + 5$ which is a straight line.

Turn over ▶

6 A function f is defined by $\mathrm{f}(x) = \dfrac{x}{\sqrt{2x-2}}$

6 (a) State the maximum possible domain of f.

[2 marks]

$2x - 2 > 0$

$2x > 2$

$x > 1$

Domain : $\{x \in \mathbb{R} : x > 1\}$

6 (b) Use the quotient rule to show that $\mathrm{f}'(x) = \dfrac{x-2}{(2x-2)^{\frac{3}{2}}}$

[3 marks]

$f(x) = \dfrac{x}{\sqrt{2x-2}}$

Let $u = x$, $v = (2x-2)^{\frac{1}{2}}$

$\dfrac{du}{dx} = 1$, $\dfrac{dv}{dx} = (2x-2)^{-\frac{1}{2}}$

$f'(x) = \dfrac{(2x-2)^{\frac{1}{2}} - x(2x-2)^{-\frac{1}{2}}}{2x-2}$

$f'(x) = \dfrac{(2x-2)^{-\frac{1}{2}}\left[(2x-2) - x\right]}{2x-2} = \dfrac{(2x-2)^{-\frac{1}{2}}(x-2)}{2x-2}$

$f'(x) = \dfrac{x-2}{(2x-2)^{3/2}}$

Do not write outside the box

6 (c) Show that the graph of $y = f(x)$ has exactly one point of inflection.

[7 marks]

At the point of inflection, $f''(x) = 0$.

$f'(x) = \frac{x-2}{(2x-2)^{3/2}}$

Let $u = x-2$, $v = (2x-2)^{3/2}$

$\frac{du}{dx} = 1$, $\frac{dv}{dx} = 3(2x-2)^{\frac{1}{2}}$

$f''(x) = \frac{(2x-2)^{3/2} - 3(x-2)(2x-2)^{\frac{1}{2}}}{(2x-2)^3} = \frac{(2x-2)^{\frac{1}{2}}[(2x-2)-3(x-2)]}{(2x-2)^3}$

$f''(x) = \frac{(2x-2)^{\frac{1}{2}}(4-x)}{(2x-2)^3}$

Set $f''(x) = 0$: $(2x-2)^{\frac{1}{2}}(4-x) = 0$

$2x-2 = 0$ or $4-x = 0$

$x = 1$ or $x = 4$

$x = 1$ is not in the domain so the point of inflection is at $x = 4$.

To check: $f''(3) = \frac{(6-2)^{\frac{1}{2}}(4-3)}{(6-2)^3} = \frac{(2)(1)}{64} = \frac{1}{32} > 0$

$f''(5) = \frac{(10-2)^{\frac{1}{2}}(4-5)}{(10-2)^3} = \frac{2\sqrt{2}(-1)}{512} = \frac{-\sqrt{2}}{256} < 0$

So the point of inflection is at $x = 4$.

6 (d) Write down the values of x for which the graph of $y = f(x)$ is convex.

[1 mark]

$1 < x < 4$

Do not write outside the box

7 (a) Given that $\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$, find y in terms of a.

[4 marks]

$\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$

$\log_a y - 2\log_a 7 - \log_a 4 = \frac{1}{2}$

$\log_a y - \log_a 7^2 - \log_a 4 = \frac{1}{2}$

$\log_a y - (\log_a 7^2 + \log_a 4) = \frac{1}{2}$

$\log_a \left(\frac{y}{7^2 \times 4}\right) = \frac{1}{2}$

$\frac{y}{196} = a^{\frac{1}{2}}$

$y = 196a^{\frac{1}{2}}$

$y = 196\sqrt{a}$

Do not write outside the box

7 (b) When asked to solve the equation

$$2\log_a x = \log_a 9 - \log_a 4$$

a student gives the following solution:

$$2\log_a x = \log_a 9 - \log_a 4$$

$$\Rightarrow 2\log_a x = \log_a \frac{9}{4}$$

$$\Rightarrow \log_a x^2 = \log_a \frac{9}{4}$$

$$\Rightarrow x^2 = \frac{9}{4}$$

$$\therefore x = \frac{3}{2} \text{ or } -\frac{3}{2}$$

Explain what is wrong with the student's solution.

[1 mark]

$x = -\frac{3}{2}$ is not a valid solution because you cannot do $\log_a\left(-\frac{3}{2}\right)$.

Turn over for the next question

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8 (a) Prove the identity $\dfrac{\sin 2x}{1+\tan^2 x} \equiv 2\sin x\cos^3 x$

[3 marks]

$\text{LHS} = \dfrac{\sin 2x}{1+\tan^2 x} = \dfrac{2\sin x\cos x}{1+\tan^2 x}$

$= \dfrac{2\sin x\cos x}{\sec^2 x}$

$= 2\sin x\cos^3 x = \text{RHS}$

Do not write outside the box

8 (b) Hence find $\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,d\theta$

[6 marks]

$$\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,d\theta = 4\int \frac{\sin 4\theta}{1+\tan^2 2\theta}\,d\theta$$

$$= 4\int 2\sin 2\theta\cos^3 2\theta\,d\theta$$

$$= 8\int \sin 2\theta\cos^3 2\theta\,d\theta$$

Let $u = \cos 2\theta$, then $\frac{du}{d\theta} = -2\sin 2\theta \Rightarrow d\theta = \frac{-1}{2\sin 2\theta}\,du$.

Making the substitutions:

$$8\int \sin 2\theta\cos^3 2\theta\,d\theta = 8\int \sin 2\theta \cdot u^3 \times \frac{-1}{2\sin 2\theta}\,du$$

$$= -4\int u^3\,du$$

$$= -4\left[\frac{1}{4}u^4\right]$$

$$= -u^4 + c$$

$$= -\cos^4 2\theta + c$$

Turn over ▶

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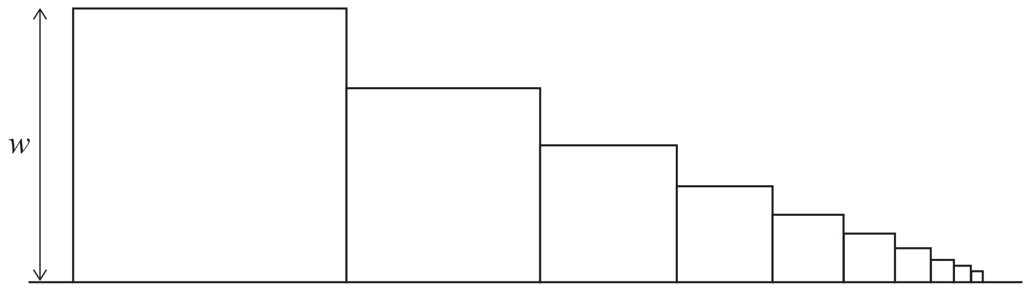
9 Helen is creating a mosaic pattern by placing square tiles next to each other along a straight line.



The area of each tile is half the area of the previous tile, and the sides of the largest tile have length w centimetres.

9 (a) Find, in terms of w, the length of the sides of the second largest tile.

[1 mark]

Area of first tile $= w^2$

Area of second tile $= \frac{w^2}{2}$

Length of second tile $= \sqrt{\frac{w^2}{2}} = \frac{w}{\sqrt{2}}$.

9 (b) Assume the tiles are in contact with adjacent tiles, but do not overlap.

Show that, no matter how many tiles are in the pattern, the total length of the series of tiles will be less than $3.5w$.

[4 marks]

This can be modelled as a geometric series with $a = w$ and $r = \frac{1}{\sqrt{2}}$.

$S_\infty = \frac{w}{1 - \frac{1}{\sqrt{2}}} = 3.41w$

$3.41w < 3.5w$ so it will never reach $3.5w$.

9 (c) Helen decides the pattern will look better if she leaves a 3 millimetre gap between adjacent tiles.

Explain how you could refine the model used in part **(b)** to account for the 3 millimetre gap, and state how the total length of the series of tiles will be affected.

[2 marks]

Add an extra 3mm to the value of the length of each tile.

Then $r > 1$ so the sum to infinity will diverge and there

will not be a limit on the total length.

Turn over for the next question

Do not write outside the box

10 Prove by contradiction that $\sqrt[3]{2}$ is an irrational number.

[7 marks]

Assume $\sqrt[3]{2}$ is rational, so $\sqrt[3]{2} = \frac{p}{q}$

where p and q have no common factors so the fraction is in its most simplified form.

$\sqrt[3]{2} = \frac{p}{q} \Rightarrow q\sqrt[3]{2} = p$

$\Rightarrow 2q^3 = p^3 \Rightarrow p$ is even

So, since p is even, let $p = 2a$. Then

$2q^3 = (2a)^3$

$2q^3 = 8a^3$

$q^3 = 4a^3 \Rightarrow q$ is even.

We assumed that p and q have no common factors, but we have now shown they are both divisible by 2.

Hence we have a contradiction, so $\sqrt[3]{2}$ cannot be irrational and therefore must be irrational.

Section B

Answer **all** questions in the spaces provided.

11 The table below shows the probability distribution for a discrete random variable X.

x	1	2	3	4	5
$P(X = x)$	k	$2k$	$4k$	$2k$	k

Find the value of k.

Circle your answer.

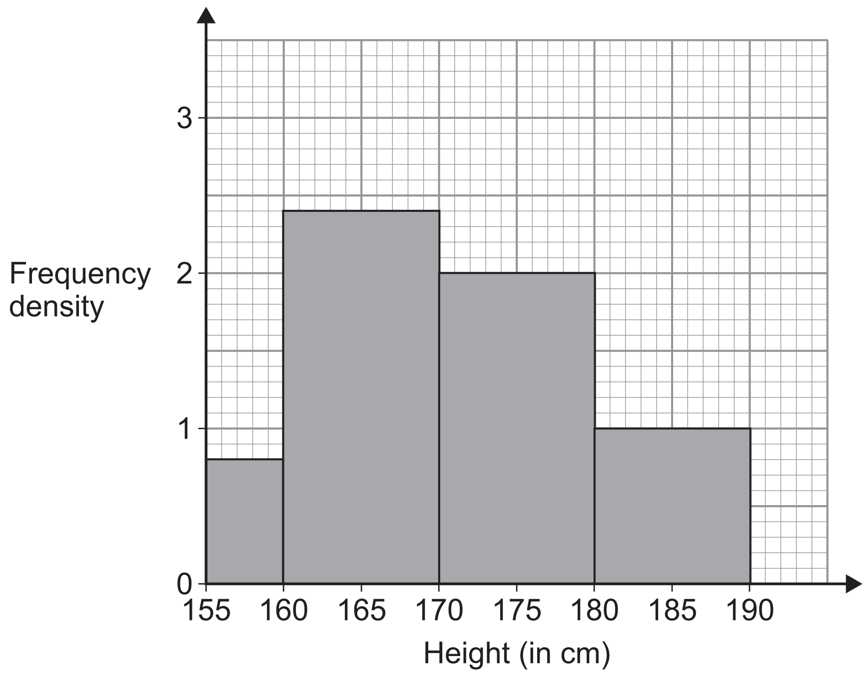
[1 mark]

$\frac{1}{2}$ $\qquad$ $\frac{1}{4}$ $\qquad$ $\frac{1}{10}$ $\qquad$ 1

$k + 2k + 4k + 2k + k = 1 \Rightarrow 10k = 1 \Rightarrow k = \frac{1}{10}$

Turn over for the next question

12 The histogram below shows the heights, in cm, of male A-level students at a particular school.



Which class interval contains the median height?

Circle your answer.

[1 mark]

[155, 160)   [160, 170)   [170, 180)   [180, 190]

Do not write outside the box

13 The table below shows an extract from the Large Data Set.

Year	**2011**	**2012**	**2013**	**2014**	**% change since 2011**
Other takeaway food brought home	0	0	0	0	−29

Sarah claims that the −29% change since 2011 is incorrect, as there is no change between 2011 and 2014.

Using your knowledge of the Large Data Set to justify your answer, explain whether Sarah's claim is correct.

[3 marks]

The values are rounded to the nearest integer so are not actually equal to zero.
If you used the unrounded numbers you could get a change of −29%, so Sarah is incorrect.

Turn over for the next question

Do not write outside the box

14 A teacher in a college asks her mathematics students what other subjects they are studying.

She finds that, of her 24 students:

12 study physics
8 study geography
4 study geography and physics

14 (a) A student is chosen at random from the class.

Determine whether the event 'the student studies physics' and the event 'the student studies geography' are independent.

[2 marks]

Let P be the event the student studies physics.

Let G be the event the student studies geography.

$P(P) = \frac{12}{24} = \frac{1}{2}$, $P(G) = \frac{8}{24} = \frac{1}{3}$.

$P(P \cap G) = \frac{4}{24} = \frac{1}{6}$

$P(P) \times P(G) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = P(P \cap G)$ so they are independent.

Do not write outside the box

14 (b) It is known that for the whole college:

the probability of a student studying mathematics is $\frac{1}{5}$

the probability of a student studying biology is $\frac{1}{6}$

the probability of a student studying biology given that they study mathematics is $\frac{3}{8}$

Calculate the probability that a student studies mathematics or biology or both.

[4 marks]

Let M be the event the student studies mathematics.

Let B be the event the student studies biology.

$P(M \cap B) = P(M) \times P(B|M)$

$= \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$

$P(M \cup B) = P(M) + P(B) - P(M \cap B)$

$= \frac{1}{5} + \frac{1}{6} - \frac{3}{40}$

$= \frac{7}{24}$

Turn over for the next question

Do not write outside the box

15 Abu visits his local hardware store to buy six light bulbs.

He knows that 15% of all bulbs at this store are faulty.

15 (a) State a distribution which can be used to model the number of faulty bulbs he buys.

[1 mark]

$B(6, 0.15)$

15 (b) Find the probability that all of the bulbs he buys are faulty.

[1 mark]

$0.15^6 = 0.0000114$

15 (c) Find the probability that at least two of the bulbs he buys are faulty.

[2 marks]

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - 0.7764$

$= 0.224$

15 (d) Find the mean of the distribution stated in part **(a)**.

[1 mark]

$6 \times 0.15 = 0.9$

15 (e) State two necessary assumptions in context so that the distribution stated in part **(a)** is valid.

[2 marks]

- A lightbulb being faulty is independent of whether or not the other light bulbs are faulty.
- The probability of a light bulb being faulty is constant.

Turn over for the next question

Do not write outside the box

16 A survey of 120 adults found that the volume, X litres per person, of carbonated drinks they consumed in a week had the following results:

$$\sum x = 165.6 \qquad \sum x^2 = 261.8$$

16 (a) (i) Calculate the mean of X.

[1 mark]

$\frac{165.6}{120} = 1.38$

16 (a) (ii) Calculate the standard deviation of X.

[2 marks]

$\sqrt{\frac{261.8}{120} - 1.38^2} = 0.52656\ldots = 0.527$

16 (b) Assuming that X can be modelled by a normal distribution find

16 (b) (i) $P(0.5 < X < 1.5)$

[2 marks]

$P(0.5 < X < 1.5)$

$= P\left(\frac{0.5-1.38}{0.527} < \frac{X-1.38}{0.527} < \frac{1.5-1.38}{0.527}\right)$

$= P(-1.67 < X < 0.23)$

$= 0.5901 - (1 - 0.9525)$

$= 0.5426$

Do not write outside the box

16 (b) (ii) P($X = 1$)

[1 mark]

0

16 (c) Determine with a reason, whether a normal distribution is suitable to model this data.

[2 marks]

$1.38 - 3(0.527) = 1.38 - 1.57968 = -0.1998$

This is less than 0 so the model may not be suitable.

16 (d) It is known that the volume, Y litres per person, of energy drinks consumed in a week may be modelled by a normal distribution with standard deviation 0.21

Given that P($Y > 0.75$) $= 0.10$, find the value of μ, correct to three significant figures.

[4 marks]

$Y \sim N(\mu, 0.21^2)$

$P(Y > 0.75) = P\left(Z > \frac{0.75-\mu}{0.21}\right) = 0.1$

$1.2816 = \frac{0.75-\mu}{0.21}$

$\mu = 0.481$

Turn over ▶

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17 Suzanne is a member of a sports club.

For each sport she competes in, she wins half of the matches.

17 (a) After buying a new tennis racket Suzanne plays 10 matches and wins 7 of them.

Investigate, at the 10% level of significance, whether Suzanne's new racket has made a difference to the probability of her winning a match.

[7 marks]

$H_0: \ p = 0.5$

$H_1: \ p \neq 0.5$

Let $X \sim B(10, 0.5)$ where X is the number of games she wins.

$P(X \geq 7) = 1 - P(X \leq 6)$

$= 1 - 0.8281$

$= 0.172$

$0.172 > 0.05$ so accept H_0. Insufficient evidence to say the new racket makes a difference.

Do not write outside the box

17 (b) After buying a new squash racket, Suzanne plays 20 matches. Find the minimum number of matches she must win for her to conclude, at the 10% level of significance, that the new racket has improved her performance.

[5 marks]

$Y \sim B(20.\ 0.5)$

$P(Y \geq 13) = 0.1316$

$P(Y \geq 14) = 0.0577$

$0.0577 > 0.1$ so she needs to win at least 14 matches to conclude she has improved.

Turn over for the next question

18 In a region of England, the government decides to use an advertising campaign to encourage people to eat more healthily.

Before the campaign, the mean consumption of chocolate per person per week was known to be 66.5 g, with a standard deviation of 21.2 g

18 (a) After the campaign, the first 750 available people from this region were surveyed to find out their average consumption of chocolate.

18 (a) (i) State the sampling method used to collect the survey.

[1 mark]

Opportunistic sampling

18 (a) (ii) Explain why this sample should not be used to conduct a hypothesis test.

[1 mark]

The sample is not random.

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18 (b) A second sample of 750 people revealed that the mean consumption of chocolate per person per week was 65.4 g

Investigate, at the 10% level of significance, whether the advertising campaign has decreased the mean consumption of chocolate per person per week.

Assume that an appropriate sampling method was used and that the consumption of chocolate is normally distributed with an unchanged standard deviation.

[6 marks]

$H_0: \mu = 66.5$

$H_1: \mu < 66.5$

$$z = \frac{65.4 - 66.5}{\left(\frac{21.2}{\sqrt{750}}\right)} = -1.42$$

The critical value for 10% is -1.28.

$-1.42 < -1.28$ so reject H_0. There is sufficient evidence that the advertising campaign reduced chocolate consumption.

END OF QUESTIONS

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